

Neural Network 2

Training 중에 일어나는 일

- Given (\mathbf{x}^i, y^i) ($i=1\dots m$)
- Define Hypothesis $H_{\theta}(\mathbf{x})$ for predicting y^j from new \mathbf{x}^j
- Choose cost function $J(\theta)$ (θ_i $i=1\dots n$) such that
- By minimizing $J(\theta)$ for **fixed** (\mathbf{x}^i, y^i) ($i=1\dots m$)
- We obtain θ for best $H_{\theta}(\mathbf{x})$

Linear regression (univariate 경우)

Hypothesis: $h_{\theta}(x) = \theta_0 + \theta_1 x$

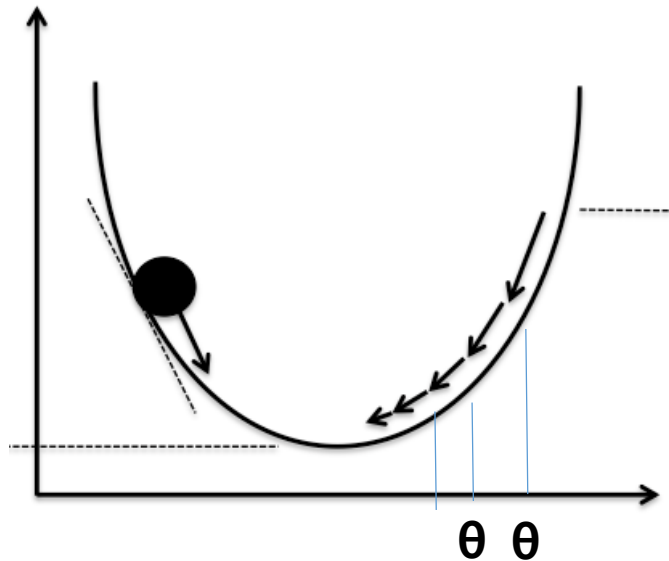
Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$

Goal: minimize $J(\theta_0, \theta_1)$
 θ_0, θ_1

최적화 알고리즘 (gradient descent)

- Minimize $J(\theta)$ for **fixed** (x^i, y^i) ($i=1\dots m$) 하려면
- 어떤 θ 에 대해서도 $J(\theta)$ 과 $\frac{dJ(\theta)}{d\theta}$ 를 알 수 있는 방법이 필요



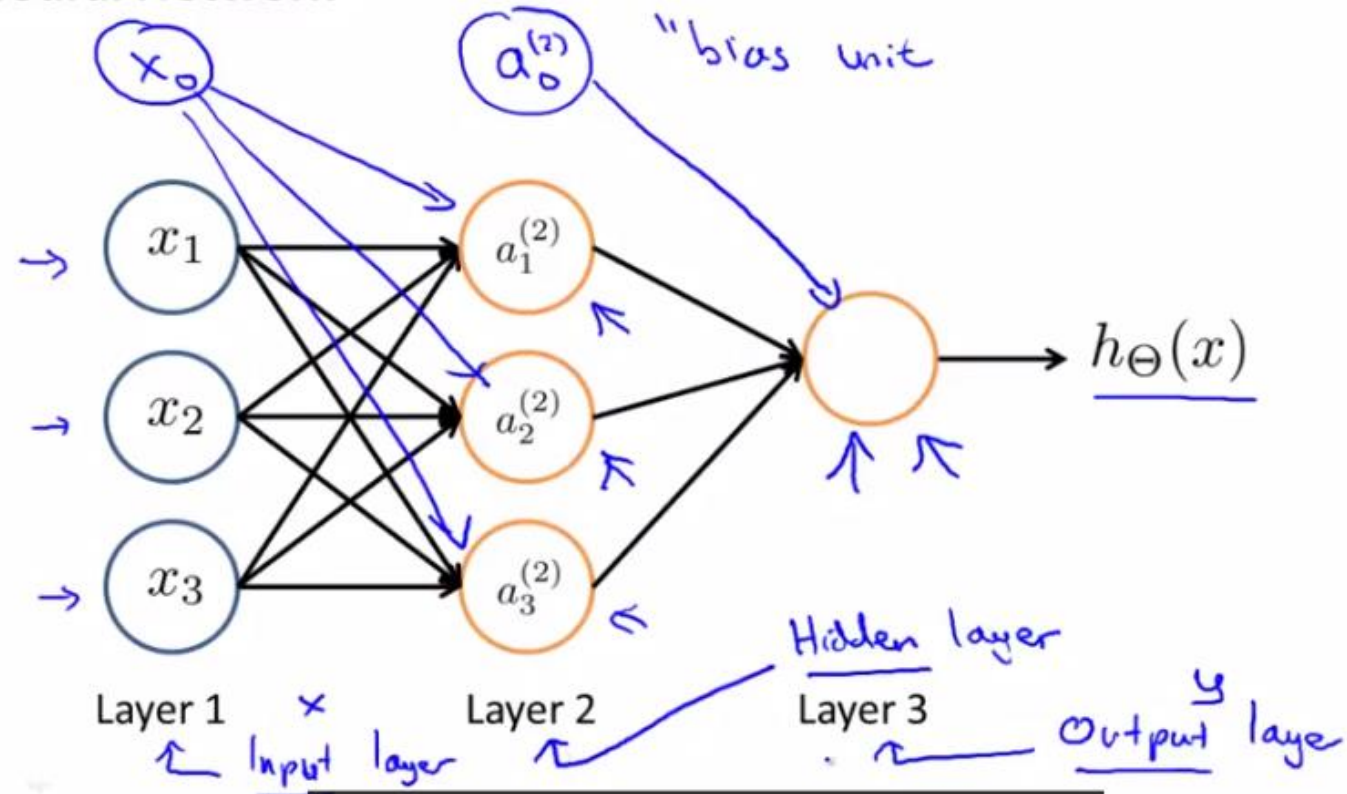
- (1) 주어진 θ 에 대해 $J(\theta)$, $\frac{dJ(\theta)}{d\theta}$ 계산
- (2) 새로운 θ 계산 (아래 공식 참조)
- (3) 새로운 $J(\theta)$ 계산하고 더 나으면 (1)로 돌아가 반복

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

learning rate

Model

Neural Network



$$\Rightarrow a_1^{(2)} = g(\Theta_{10}^{(1)} x_0 + \Theta_{11}^{(1)} x_1 + \Theta_{12}^{(1)} x_2 + \Theta_{13}^{(1)} x_3)$$

$$\Rightarrow a_2^{(2)} = g(\Theta_{20}^{(1)} x_0 + \Theta_{21}^{(1)} x_1 + \Theta_{22}^{(1)} x_2 + \Theta_{23}^{(1)} x_3)$$

$$\Rightarrow a_3^{(2)} = g(\Theta_{30}^{(1)} x_0 + \Theta_{31}^{(1)} x_1 + \Theta_{32}^{(1)} x_2 + \Theta_{33}^{(1)} x_3)$$

$$h_{\Theta}(x) = a_1^{(3)} = g(\Theta_{10}^{(2)} a_0^{(2)} + \Theta_{11}^{(2)} a_1^{(2)} + \Theta_{12}^{(2)} a_2^{(2)} + \Theta_{13}^{(2)} a_3^{(2)})$$

$\Theta^{(2)}$

↓

Neural network

- 레이어간 weight matrix들이 바로 θ
- 주어진 모든 θ_{jk}^l 에 대해서 $J(\theta)$ 과 $\frac{\partial J(\theta)}{\partial \theta_{jk}^l}$ 값을.. 알 수 있나?
- $J(\theta)$: (x^i, y^i) ($i=1\dots m$) 에 대해서 y^i 와 NN의 예측의 차이
→ forward propagation for **ALL, fixed (x^i, y^i) ($i=1\dots m$)**
- $\frac{\partial J(\theta)}{\partial \theta_{jk}^l}$: NN 의 결과 예측값에 미치는 θ_{jk}^l 의 영향의 크기
→ backpropagation for **ALL, fixed (x^i, y^i) ($i=1\dots m$)**

NN training 알고리즘

1. Randomly initialize the weights

2. Implement forward propagation to get $h_{\Theta}(x^{(i)})$ for any $x^{(i)}$

3. Implement the **cost function**

$$J(\Theta) = -\frac{1}{m} \sum_{i=1}^m \sum_{k=1}^K \left[y_k^{(i)} \log((h_{\Theta}(x^{(i)}))_k) + (1 - y_k^{(i)}) \log(1 - (h_{\Theta}(x^{(i)}))_k) \right] + \frac{\lambda}{2m} \sum_{l=1}^{L-1} \sum_{i=1}^{s_l} \sum_{j=1}^{s_{l+1}} (\Theta_{j,i}^{(l)})^2$$

4. Implement backpropagation to compute **partial derivatives**

5. Use gradient checking to confirm that your backpropagation works. Then disable gradient checking.

6. Use gradient descent or a built-in optimization function to minimize the cost function with the weights in theta.

When we perform forward and back propagation, we loop on every training example:

```
1 for i = 1:m,  
2     Perform forward propagation and backpropagation using example (x(i),y(i))  
3     (Get activations a(l) and delta terms d(l) for l = 2,...,L)
```

Backpropagation algorithm

→ Training set $\{(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})\}$

Set $\Delta_{ij}^{(l)} = 0$ (for all l, i, j).

(used to compute $\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta)$)

For $i = 1$ to $m \leftarrow (x^{(i)}, y^{(i)})$.

Set $a^{(1)} = x^{(i)}$

→ Perform forward propagation to compute $a^{(l)}$ for $l = 2, 3, \dots, L$

→ Using $y^{(i)}$, compute $\delta^{(L)} = a^{(L)} - y^{(i)}$

→ Compute $\delta^{(L-1)}, \delta^{(L-2)}, \dots, \delta^{(2)}$

→ $\Delta_{ij}^{(l)} := \Delta_{ij}^{(l)} + a_j^{(l)} \delta_i^{(l+1)}$

$\Delta^{(l)} := \Delta^{(l)} + \delta^{(l+1)} (a^{(l)})^T$

→ $D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)} + \lambda \Theta_{ij}^{(l)}$ if $j \neq 0$

→ $D_{ij}^{(l)} := \frac{1}{m} \Delta_{ij}^{(l)}$ if $j = 0$

$$\frac{\partial}{\partial \Theta_{ij}^{(l)}} J(\Theta) = D_{ij}^{(l)}$$

$$\delta^{(l)} = ((\Theta^{(l)})^T \delta^{(l+1)}) .* a^{(l)} .* (1 - a^{(l)})$$

Intuition

$$g'(z^{(l)}) = a^{(l)} .* (1 - a^{(l)})$$

Back propagation (chain rule)

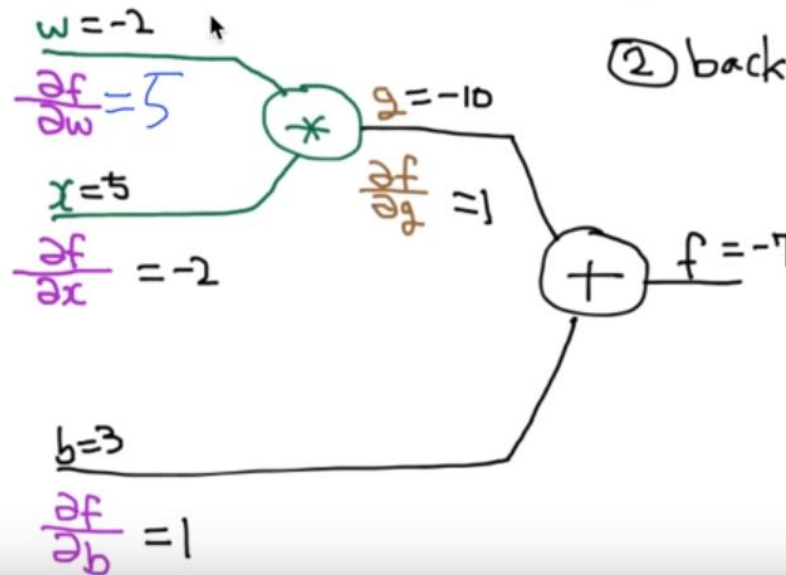
$$f = wx + b, \quad g = wx, \quad f = g + b$$

$\frac{\partial f}{\partial g} = 1, \quad \frac{\partial f}{\partial b} = 1$
 $\frac{\partial g}{\partial w} = x, \quad \frac{\partial g}{\partial x} = w$

① forward ($w = -2, x = 5, b = 3$)

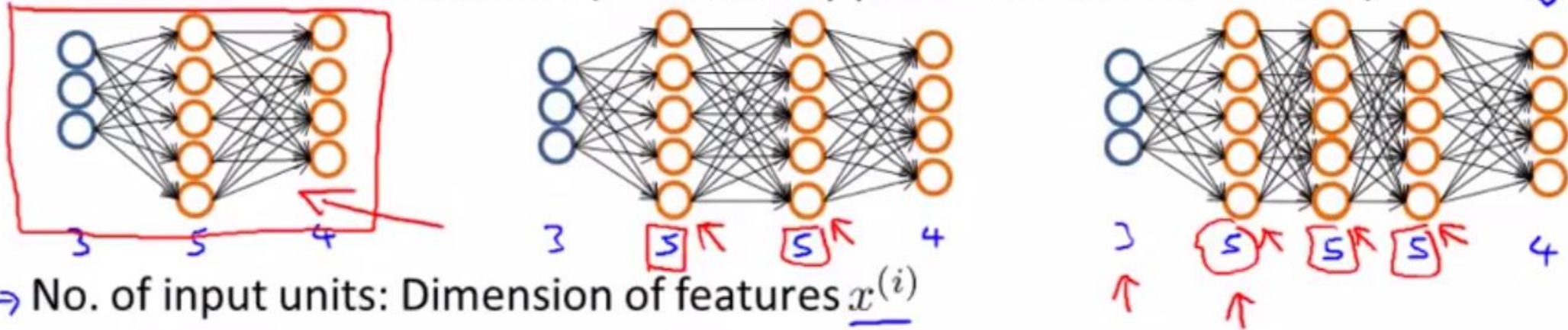
② backward

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial g} \frac{\partial g}{\partial x} = 1 * w = -2$$
$$\frac{\partial f}{\partial w} = \frac{\partial f}{\partial g} \frac{\partial g}{\partial w} = 1 * x = 5$$



Others – choosing network shape

Pick a network architecture (connectivity pattern between neurons)



→ No. of input units: Dimension of features $x^{(i)}$

→ No. output units: Number of classes

Reasonable default: 1 hidden layer, or if >1 hidden layer, have same no. of hidden units in every layer (usually the more the better)

Others

- ReLU() is better than sigmoid
- Why randomize the initialize θ_{jk}^l ?
- Dropout option
- <https://www.coursera.org/learn/machine-learning/lecture/zYS8T/autonomous-driving>

실습

- <http://machinelearningmastery.com/tutorial-first-neural-network-python-keras/>
- <http://machinelearningmastery.com/setup-python-environment-machine-learning-deep-learning-anaconda/>